# Limit State Material Manual

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## 1. Introduction

This manual is a complement to the example of a column that implements Limit State Material (http://opensees.berkeley.edu/wiki/index.php/LimitStateMaterialExampleDebugged). The example demonstrates analytical prediction of the lateral drifts at the shear and axial failure of a rectangular reinforced concrete column tested by Elwood (2004). Although parameters of the Limit State Material within the example were defined based on the experimental data, the manual provides a procedure that can be used to calculate these parameters analytically (based on Elwood, 2004). In addition to describing a model for predicting axial and shear failure of the column, the author of the manual has performed a parametric study considering different axial load and transverse reinforcement ratios to show the effect of these parameters on the drifts at shear and the axial failure.

## 2. Column geometry and analytical model

### 2.1 Column geometry

The column used in this study is 58 in. tall. It is fully fixed at both ends. The cross-section is square (9 × 9 in.). It is reinforced with total of 8 longitudinal bars (four #5 bars and four #4 bars) giving the longitudinal reinforcement ratio of 2.52% (Figure 1). The transverse reinforcement ratio is 0.18% (W2.9 wire @ 7.2” o.c) (Figure 1). The concrete cover is 1 inch on all sides of the section.

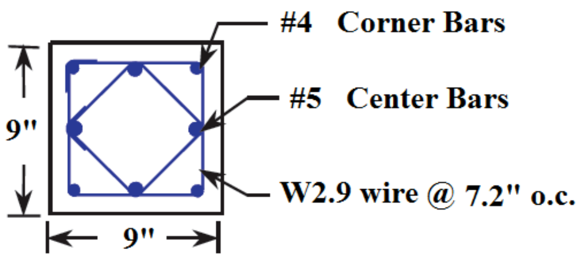


**Figure 1** – Column Cross-Section

### 2.2 Column model

The column is modeled with a force-based beam-column element with fiber sections. Concrete01 material is used to define concrete fibers. For the purpose of this example the confinement effects of transverse reinforcement on the concrete core were not accounted for. Steel02 material is used to define reinforcing steel fibers. Input parameters for Concrete01 and Steel02 are given in the Appendix of this document.

To account for strain penetration of longitudinal reinforcement, elastic rotational spring is defined at each column end (Figure 2 – Sketches A and B). Rotational springs are defined using zero length elements. The rotational stiffness of the spring is calculated following recommendations of Elwood and Eberhard (2008):

$$K_{slip} = \frac{8u}{d_b f_s} EI_{flex} \qquad (MPa)$$

Where, $u$ is the bond stress (assumed to be $0.8\sqrt{f_c'}$)(Ref.5), $d_b$ is the nominal diameter of the longitudinal reinforcement, $f_s$ is the yield tensile stress in the longitudinal reinforcement, and $EI_{flex}$ is the effective flexural stiffness. The effective flexural stiffness is calculated from moment-curvature analysis of a column section. For this specific section it is $0.4EI_{gross}$, where $E$ is the concrete modulus of elasticity and $I_{gross}$ is the gross section moment of inertia.

To capture column shear and axial strength degradation, axial and shear springs accompany rotational spring at the top of the column (Figure 2 – Sketch B). They are defined at the top of the column as experimental results presented in Elwood (2004) demonstrated that shear and axial failure occurred at the top of the column. Axial spring is defined with the limit state material and the axial limit curve assigned to it.  Shear spring is defined with the limit state material and the shear limit curve assigned to it. Input parameters of the limit state material, shear and axial limit curves are given in the Appendix of this document.

To provide transition of the axial force to the column very soft axial spring is defined at the top of the column (Figure 2 – Sketch C).

All spring elements are modeled using zeroLenght elements of OpenSees. To assure the stability of the model horizontal translation and rotation of nodes 2 and 4 are fixed.

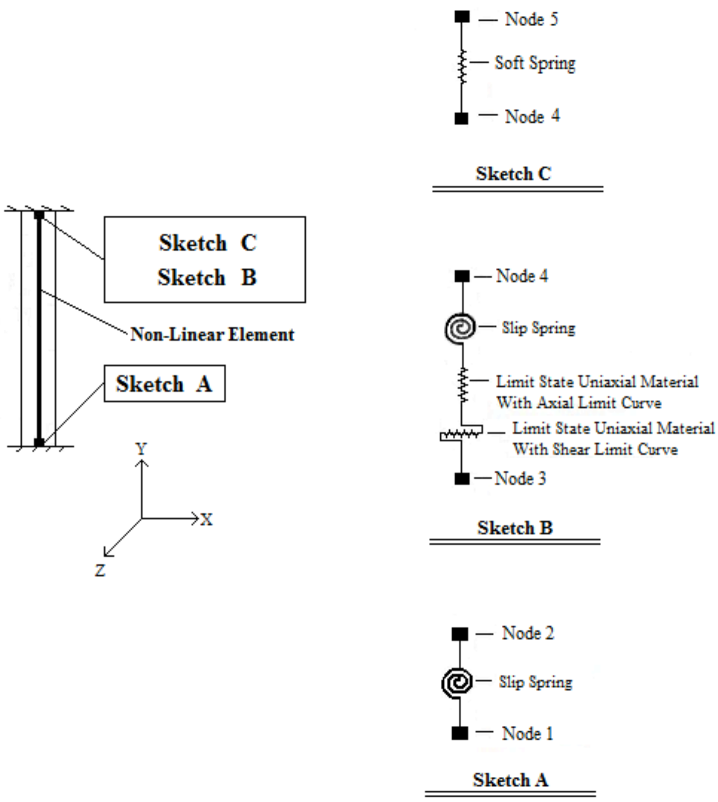


**Figure 2** – Schematic presentation of the column model.

## 3. Calculation of parameters for the limit state material and limit curves

To define axial and shear limit curves it is important to define the slope of the third branch in the post-failure backbone curve, $K_{deg}$ (Figure 3). Calculation of $K_{deg}$ is described hereafter.

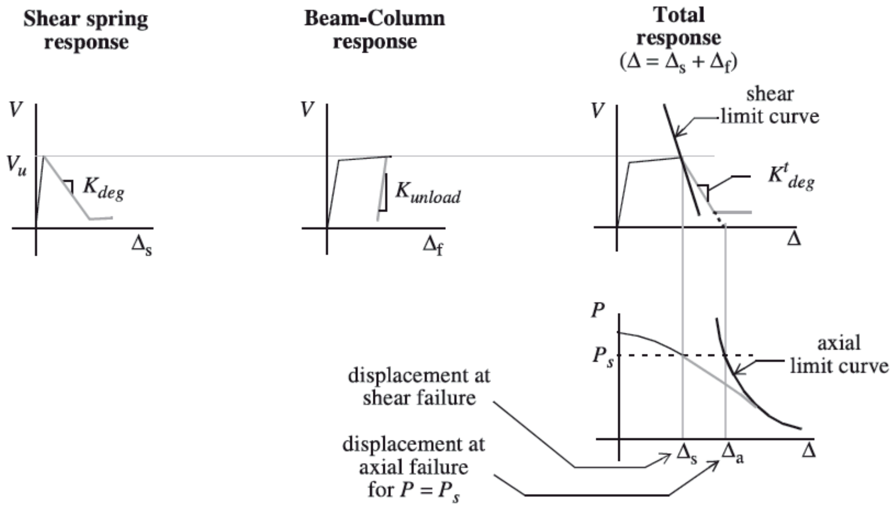


**Figure 3** – Schematics for $K_{deg}$

Experimental studies have shown that axial failure tends to occur when the shear strength degrades to approximately zero (Nakamura and Yoshimura 2002). Hence, $K_{deg}$ can be estimated using the calculated drift at the axial failure as illustrated in Figure 3. When shear failure is detected (the intersection of the total response and the shear limit curve) the degrading slope for the total response, $K^t_{deg}$, can be estimated as follows (Elwood 2004):

$$K^t_{deg} = \frac{V_u}{\Delta_a - \Delta_s} \qquad (1)$$

where $V_u$ is the ultimate shear capacity of the column, $\Delta_s$ is the displacement at shear failure, and $\Delta_a$ is the displacement at axial failure for the axial load at the time of shear failure, $P_s$ (Elwood, 2004). Displacements at the shear and axial failure can be calculated following Equations 2 or 3 and 4, respectively.

$$\left(\frac{\Delta_s}{L}\right) = \frac{3}{100} + 4\rho'' - \frac{1}{40}\frac{v}{\sqrt{f'_c}} - \frac{1}{40}\frac{P}{A_g.f'_c} \geq \frac{1}{100} \qquad (MPa) \qquad (2)$$

$$\left(\frac{\Delta_s}{L}\right) = \frac{3}{100} + 4\rho'' - \frac{1}{500}\frac{v}{\sqrt{f'_c}} - \frac{1}{40}\frac{P}{A_g.f'_c} \geq \frac{1}{100} \qquad (psi) \qquad (3)$$

$$\left(\frac{\Delta_a}{L}\right) = \frac{4}{100}\frac{1 + (tan\theta)^2}{tan\theta + P(\frac{s}{A_{st}F_{yt}d_c tan\theta})} \qquad (MPa)\ or\ (psi) \qquad (4)$$

In the given equations $(\Delta_s/L)$ is the drift ratio at shear failure, $\rho''$ is the transverse reinforcement ratio, $v$ is the nominal shear stress, $f'_c$ is the concrete compressive strength, $P$ is the axial load on column at shear failure, $A_g$ is the gross cross-sectional area, $(\Delta_a/L)$ is the drift ratio at axial failure, $\theta$ is the critical crack angle from the horizontal (assumed to be 65°), $s$ is the spacing of the transverse reinforcement, $A_{st}$ is the area of the transverse reinforcement, $F_{yt}$ is the yield strength of the transverse reinforcement. For the Limit State Material to work properly, shear limit curve needs to be defined in units of (psi) or (MPa).

Since the shear spring and the beam-column element are in series, the total flexibility is equal to the sum of the flexibilities of the shear spring and the beam-column element. Hence, $K_{deg}$ can be determined as follows:

$$K_{deg} = \left(\frac{1}{K_{deg}^{t}} - \frac{1}{K_{unload}}\right)^{-1} \qquad (5)$$

where $K_{unload}$ is the unloading stiffness of the beam-column element. It depends on the boundary conditions of the column (e.g., for a cantilever column $K_{unload}$ is $3EI_{flex}/L^3$ where, $EI_{flex}$ is effective flexural stiffness and $L$ is the height of column).

### 3.1 Calculation of Kdeg

To calculate $K_{deg}$, the ultimate shear capacity, $V_u$, of the column must be calculated first.

$$V_u = V_c + V_s \qquad (6)$$

where, $V_c$ is the concrete contribution to shear strength and $V_s$ is the steel contribution to shear strength.
Concrete contribution to shear strength, $V_c$, may be calculated as follows:

$$V_c = k\left\{\frac{6\sqrt{f_c'}}{\frac{a}{d}}\sqrt{1+\frac{P}{6A_g\sqrt{f_c'}}}\right\}A_g \qquad (psi) \qquad (7)$$

where, $a$ is a distance from maximum moment to inflection point, $d$ is the effective depth, $P$ is the axial load, $A_g$ is the gross concrete area, and $f_c'$ is the concrete compressive strength.
The steel contribution to the shear strength, $V_s$, may be calculated as follows:

$$V_s = k\frac{A_{sw}f_y d}{s} \qquad (8)$$

Where, $k$ is a modifier that accounts for strength degradation within the flexural plastic hinges, $A_{sw}$ is the area of the transverse reinforcement, $f_y$ is the yield strength of the transverse reinforcement, and $s$ is the spacing of the transverse reinforcement. If $D$ is displacement ductility, $k$ coefficient (Aschheim, 1993; Priestley, 1994) can be calculated following Equation 9:

$$\begin{cases} k = 1 & if \quad D \leq 2.0 \\ k = -0.075D + 1.15 & if \quad 2.0 \leq D \leq 6.0 \\ k = 0.7 & if \quad D \geq 6.0 \end{cases} \qquad (9)$$

Given the ultimate shear strength (Eq. 6), the displacement at the shear and axial failure (Eqs. 2 to 4), the degrading slope for the total response, $K^{t}_{deg}$, can be calculated following Equation 1. Given $K^{t}_{deg}$, $K_{deg}$ can be calculated based on Equation 5.

The parameters used to define the shear and axial curves in the OpenSees example of LimitStateMaterail application are calculated based on experimental data. For the comparison purposes, they are herein calculated based on the geometry of the column following the procedure described in the manual. The results are presented in Table 1. The input parameters for calculation of $K_{deg}$ are given below:

$b$=$h$=9" , $d$=7.75", $L$=58", $S$=7.2" , $Ag$=81 inch$^2$, $A_{st}$ = 0.116 inch$^2$ , $\rho$ = 0.0018, $a/d$ = 3.742,
$f'_c$ = 3.517 Kips/inch$^2$, $f_{yt}$=95 Kips/inch$^2$, $\theta$=65° , $tan\theta$ = 2.1445, $P$=70Kips , $k$ = 1, $Ec$=3400 Kips/inch$^2$

**Table 1 –** Calculation of $K_{deg}$: numerical vs. experimental

| Equation | Parameter | Unit | Analysis result | Experimental Result*** | The Difference (%) |
|---|---|---|---|---|---|
| 8 | $V_s$ | kip | 11.862 | ----- | ----- |
| 7 | $V_c$ | kip | 14.262 | ----- | ----- |
| 6 | $V_u$ | kip | 26.124 | ----- | ----- |
| * | $v$ | ksi | 0.322 | ----- | ----- |
| 3 | $\Delta_s/L$ | ----- | 2.014% | 2.098% | 4.17 |
| 4 | $\Delta_a/L$ | ----- | 4.574% | 4.612% | 0.83 |
| 1 | $K_{deg}^{t}$ | kip/in | 17.595 | ----- | ----- |
| ** | $K_{unload}$ | kip/in | 57.156 | ----- | ----- |
| 5 | $K_{deg}$ | kip/in | 25.42 | 24.7 | 2.91 |

* $v=Vu/(b*h)$

** $K_{unload} = 12EI_{eff}/L^3$, where $EI_{eff} = 0.5\ EI_g$ (ASCE/SEI 41 Concrete Provisions)

*** From experimental lateral force-lateral displacement curve

The difference between numerical and experimental value of $K_{deg}$ is 2.91%. No difference in response was observed if the analysis was performed utilizing numerically calculated value for $K_{deg}$.

## 4. Parametric study

Pushover analysis was performed on a column to determine the drifts at which shear and axial failure of the column occur. The effect of the initial axial load and the transverse reinforcement ratio on the shear and axial failure was studied. The axial load ratios ($P/A_g f_c'$) considered in these study were 0.15, 0.25, and 0.35. The transverse reinforcement ratios were 0.09%, 0.18%, and 0.36%. When the effect of the initial axial load on the shear and axial failure was studied the transverse reinforcement ratio was set to 0.18%. When the effect of the transverse reinforcement ratio on the shear and axial failure was studied the initial axial load ratio was set to 0.25.

### 4.1 The effects of axial load ratio on drifts at the shear and axial failure

According to Equations 3 & 4, the drift ratio of the column at shear and axial failure decreases with the increase of the axial load. The pushover curves accompanied with the shear and axial limit curves are shown in Figures 4 to 5 for the three values of the initial axial load imposed on the column. Table 2 summarizes the drifts at the shear and the axial failure. The effect of the initial axial load on the drifts at the shear and axial failure is shown in Figure 6.

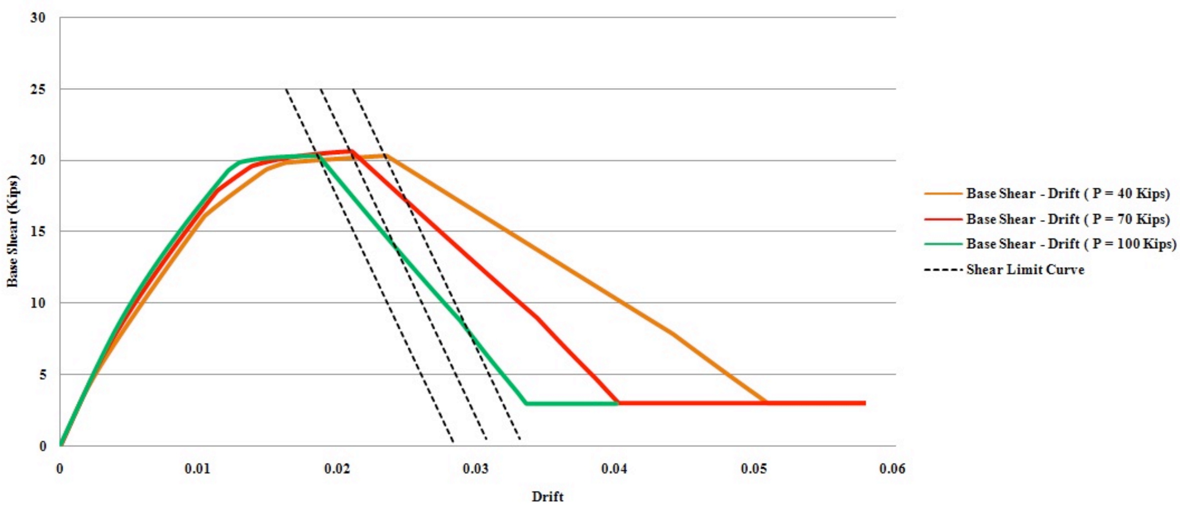


**Figure 4** – Base Shear vs. Lateral Drift

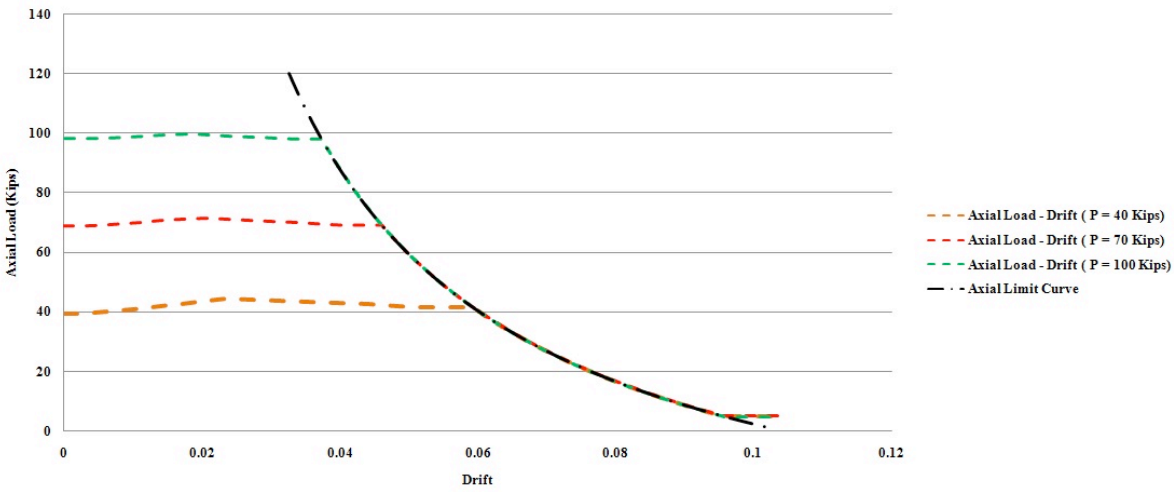


**Figure 5** – Axial Load vs. Lateral Drift

**Table 2** – The effects of the initial axial load on the drift ratios at the shear and axial failure

| TEST | ρ (%) | Initial P (Kips) | P/ $A_g.f'_c$ | $V_{max}$ (Kips) | $P_s$ (Kips) | Ps/$A_g.f'_c$ | $\Delta_s/L$ (%) | $\Delta_a/L$ (%) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.18 | 40 | 0.15 | 20.33 | 44.36 | 0.15 | 2.348 | 5.934 |
| 2 | 0.18 | 70 | 0.25 | 20.62 | 71.34 | 0.25 | 2.098 | 4.612 |
| 3 | 0.18 | 100 | 0.35 | 20.3 | 99.69 | 0.35 | 1.864 | 3.738 |

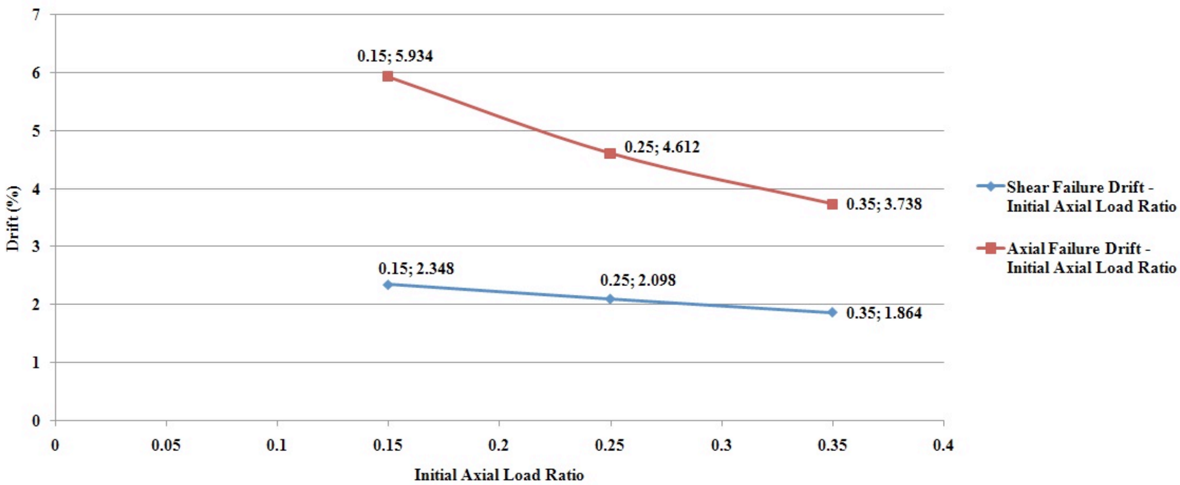


**Figure 6 –** The effect of initial axial load ratio on the drifts at the shear and axial failure

### 4.2 The effects of transverse reinforcement ratio on the drifts at the shear and axial failure

According to Equations 3 & 4, the drift ratio capacity of column at shear and axial failure increases with the increase of the axial load. The pushover curves accompanied with the shear and axial limit curves are shown in Figures 7 and 8 for the three values of the transverse reinforcement ratio. Table 3 summarizes the drifts at the shear and axial failure. The effect of the transverse reinforcement ratio on the drifts at the shear and axial failure is shown in Figure 9.

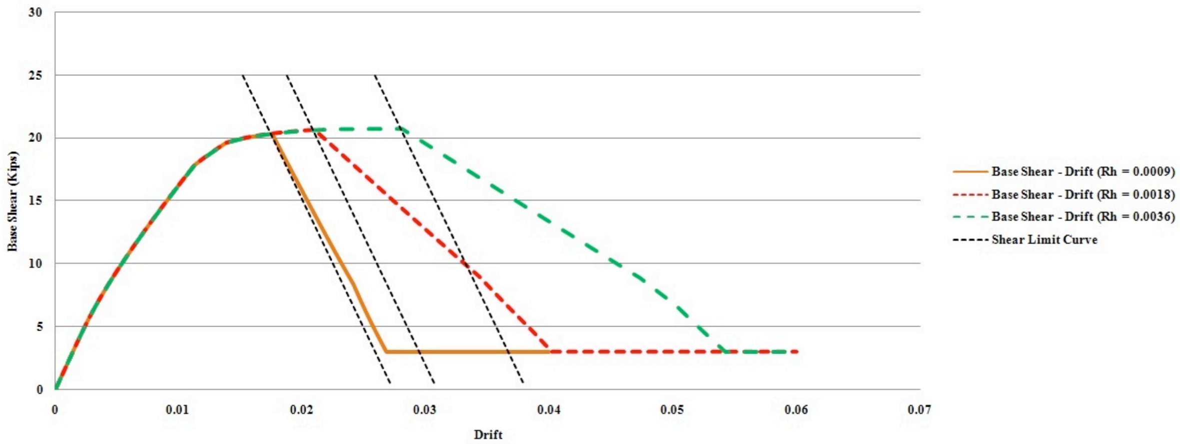


**Figure 7** – Base Shear vs. Lateral Drift

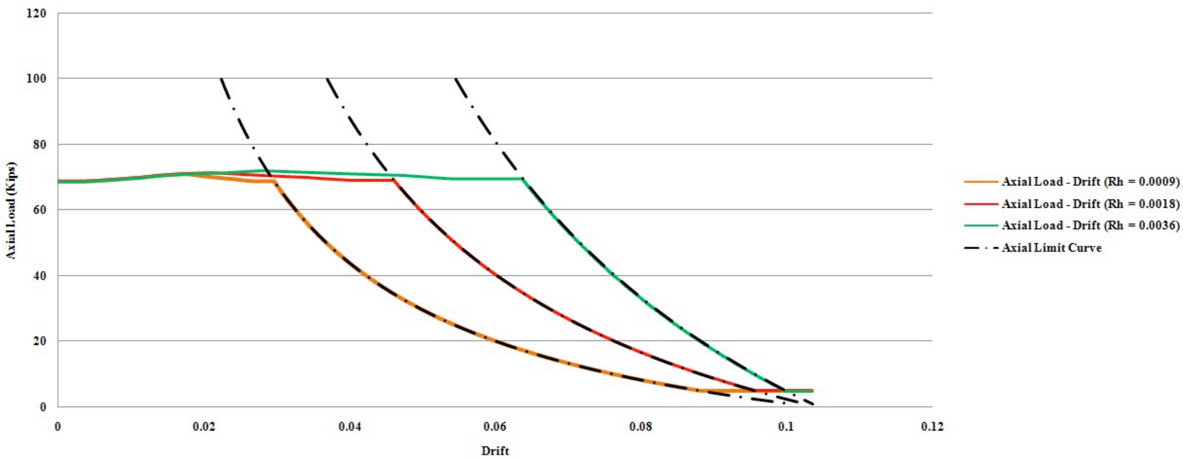


**Figure 8** – Axial Load vs. Lateral Drift

**Table 3** – The effects of transverse reinforcement ratio on the drift ratios at the shear and axial failure

| TEST | ρ (%) | $V_{max}$ (Kips) | $P_S$ (Kips) | $\Delta_s/L$ (%) | $\Delta_a/L$ (%) |
|---|---|---|---|---|---|
| 1 | 0.09 | 20.33 | 71.05 | 1.755 | 2.964 |
| 2 | 0.18 | 20.62 | 71.34 | 2.098 | 4.612 |
| 3 | 0.36 | 20.73 | 72.14 | 2.805 | 6.378 |

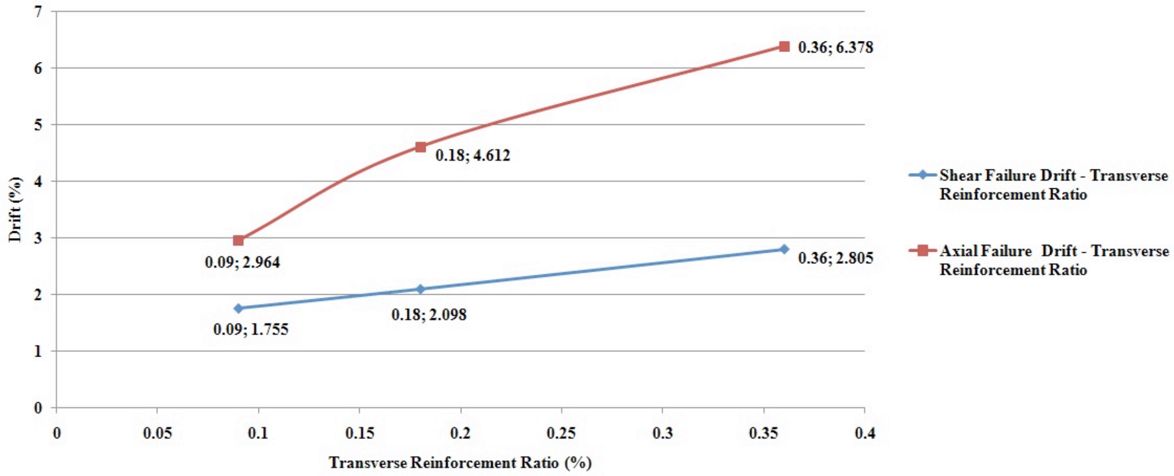


**Figure 9 –** The effect of the transverse reinforcement ratio on the drift at the shear and axial failure

## Note:

Shear failure is calculated based on shear and flexural forces when $0.7 \le V_p/V_n \le 1.0$ (flexure-shear model - shear failure following flexural yielding). If $V_p/V_n \le 0.7$ the shear failure is calculated based on flexural forces (flexure model - flexural yielding without shear failure). If $V_p/V_n \ge 1.0$ the shear failure is calculated based on shear forces (shear failure before flexural yielding) (Sezen 2002)(Figure 10).

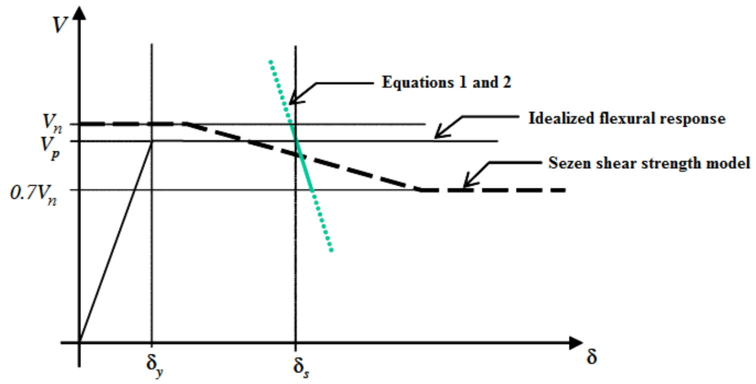


**Fig 10** – Comparison of Sezen shear strength model and Elwood model for drift ratio at shear failure

## Acknowledgment:

I gratefully appreciate the guidance in writhing of this document by my dear supervisor Dr. Mohammad KhanMohammadi and Dr. Vesna Terzic.

## References:

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6- Sezen, H, Seismic response and modeling of lightly reinforced concrete building columns, Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of California, Berkeley, 2002.

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# APPENDIX

The materials used in the example posted on the OpenSees wiki are described herein.

## 1) Concrete material:

**uniaxialMaterial Concrete01 $matTag $fpc $epsc0 $fpcu $epsU**

Where,

***$matTag*** unique material object integer tag
***$fpc*** concrete compressive strength *
***$epsc0*** concrete strain at maximum strength *
***$fpcu*** concrete crushing strength *
***$epsU*** concrete strain at crushing strength *

**NOTE:**
*Compressive concrete parameters should be input as negative values.

In this example:
**uniaxialMaterial Concrete01 $coreTag -3.517 -0.002 -3.517 -0.0052**

## 2) Steel material:

**uniaxialMaterial Hysteretic $matTag $fy $esy $fu $esu -$fy -$esy -$fu -$esu $pinchX $pinchY $damage1 $damage2 $beta $curveTag $curveType.**

Where,

***$matTag*** unique material object integer tag
***$fy*** yield stress of steel (ksi)
***$esy*** yield strain of steel – (*esy = fy/Es,* where *Es* is the modulus of elasticity of steel equal to 29000 ksi)
***$fu*** ultimate stress of steel (ksi) – (*fu = fy+ alphaS*Es*(esu-esy),* where *alphaS* is hardening ratio)
***$esu*** ultimate strain of steel
***$pinchX*** pinching factor for strain (or deformation) during reloading
***$pinchY*** pinching factor for stress (or force) during reloading
***$damage1*** damage due to ductility
***$damage2*** damage due to energy
***$beta*** power used to determine the degraded unloading stiffness based on ductility, (optional, default=0.0)

**NOTE:**
*negative backbone points should be entered as negative numeric values

In this example:

**uniaxialMaterial Hysteretic $steelTag 69.5 0.0024 133.71 0.15 -69.5 -0.0024 -133.71 -0.15 1.0 1.0\
0.0 0.0**

### 3) Shear limit curve:

**# define limit surface using shear drift model**
**limitCurve Shear $curveTag $eleTag $rho $fc $b $h $d $Fsw $Kdeg $Fres $defType $forType\
<$ndI $ndJ $dof $perpDirn $delta>**

Where,

| | |
|---|---|
| ***$curveTag*** | unique limit curve object integer tag |
| ***$eleTag*** | integer element tag for the associated beam-column element |
| ***$rho*** | transverse reinforcement ratio ($A_{st}/bS$, where, $A_{st}$ is the area of the transverse reinforcement, $b$ is column width, $S$ is the spacing of the transverse reinforcement) |
| ***$fc*** | concrete compressive strength (psi) |
| ***$b*** | column width (inch) |
| ***$h*** | full column depth (inch) |
| ***$d*** | effective column depth (inch) |
| ***$Fsw*** | floating point value describing the amount of transverse reinforcement ($F_{sw} = A_{st}f_{yt}d_c/S$) where, $A_{st}$ is the area of the transverse reinforcement, $F_{yt}$ is the yield strength of the transverse reinforcement ratio, $d_c$ is effective column depth, $S$ is the spacing of the transverse reinforcement) |
| ***$Kdeg*** | If positive: unloading stiffness of beam-column element if negative: slope of third branch of post-failure backbone (calculation of this is given latter in the document) |
| ***$Fres*** | floating point value for the residual force capacity of the post-failure backbone |
| ***$defType*** | integer flag for type of deformation defining the abscissa of the limit curve (1 = maximum beam-column chord rotations, 2 = drift based on displacement of nodes *ndI* and *ndJ*) |
| ***$forType*** | integer flag for type of force defining the ordinate of the limit curve* (0 = force in associated limit state material, 1 = shear in beam-column element) |
| ***$ndI*** | integer node tag for the first associated node (normally node I of *$eleTag* beam-column element) |
| ***$ndJ*** | integer node tag for the second associated node (normally node J of *$eleTag* beam-column element) |
| ***$dof*** | nodal degree of freedom to monitor for drift** |
| ***$perpDirn*** | perpendicular global direction from which length is determined to compute drift** |
| ***$delta*** | drift (floating point value) used to shift shear limit curve |

**NOTE**:
* Option 1 assumes no member loads.
** 1 = X, 2 = Y, 3 = Z

In this example:
**limitCurve Shear $shearCurveTag $bcTag 0.0018 3517.0 9.0 9.0 7.75 11.87 24.7 3.0 2 0\
1 4 1 2 0.0**

### 4) Axial limit curve:

**# define limit surface**
**limitCurve Axial $curveTag $eleTag $Fsw $Kdeg $Fres $defType $forType
<$ndI $ndJ $dof $perpDirn $delta>**

Where,

| | |
|---|---|
| ***$curveTag*** | unique limit curve object integer tag |
| ***$eleTag*** | integer element tag for the associated beam-column element |
| ***$Fsw*** | floating point value describing the amount of transverse reinforcement ($F_{sw} = A_{st}f_{yt}d_c/S$) |
| ***$Kdeg*** | floating point value for the slope of the third branch in the post-failure backbone curve, assumed to be negative (from experimental data it is acquired to be, |

Axial Load/Axial Displacement = -90 Kips/Inch. It can also be calculated from the effective axial stiffness of the column after axial failure ($(E_cAg)_{eff}/L$). Based on the experimental results it is shown that the *Kdeg* can be calculated as $-0.02E_cAg/L$ which for this specific column gives a value of *-94.97 Kips/Inch* and an error of 5.52% from the experimental value.

In this example:
**limitCurve Axial $axialCurveTag $bcTag 11.87 -90.0 5.0 2 2**
**1 4 1 2 0.0 0**

**5) Limit state material command:**

**# define Limit State Material**
**uniaxialMaterial LimitState $matTag $s1p $e1p $s2p $e2p $s3p $e3p $s1n $e1n $s2n $e2n $s3n $e3n**
**$pinchX $pinchY $damage1 $damage2 $beta $curveTag $curveType.**

Where,
***$matTag*** unique material object integer tag
***$s1p $e1p*** stress and strain (or force & deformation) at first point of the envelope in the positive direction
***$s2p $e2p*** stress and strain (or force & deformation) at second point of the envelope in the positive direction
***$s3p $e3p*** stress and strain (or force & deformation) at third point of the envelope in the positive direction (optional)
***$s1n $e1n*** stress and strain (or force & deformation) at first point of the envelope in the negative direction*
***$s2n $e2n*** stress and strain (or force & deformation) at second point of the envelope in the negative direction*
***$s3n $e3n*** stress and strain (or force & deformation) at third point of the envelope in the negative direction (optional)*
***$pinchX*** pinching factor for strain (or deformation) during reloading
***$pinchY*** pinching factor for stress (or force) during reloading
***$damage1*** damage due to ductility
***$damage2*** damage due to energy
***$beta*** power used to determine the degraded unloading stiffness based on ductility, (optional, default=0.0)
***$curveTag*** an integer tag for the LimitCurve defining the limit surface
***$curveType*** an integer defining the type of LimitCurve (0 = no curve, 1 = axial curve, all other curves can be any other integer)

**NOTE:**
*negative backbone points should be entered as negative numeric values

**Shear limit curve:**

rigidSlope = 1700 Kips/inch ($=GA_g/L$, Where $G$ is the shear modulus, $A_g$ is the gross cross-sectional area and $L$ is the height of column).

The shear strengths are selected to be:
Vi1 = 25.0 Kips
Vi2 = 30.0 Kips
Vi3 = 45.0 Kips

To develop the shear spring response (Fig. 3), the shear forces *Vi* (*i*=1 to 3) need to be assigned values in the increasing order as they all lay on the same line. It is also important that the last shear strength, *Vi3*, is greater than the ultimate shear strength, *Vu* (Eq.6). Even if the value of *Vi3* is significantly higher than *Vu*, the results of the analysis will not change. In this example the value of *Vi3/Vu* is 45/26.12 = 1.72.

In this example:
**uniaxialMaterial LimitState $shearTag**
**$Vi1 [expr $Vi1/$rigidSlope] $Vi2 [expr $Vi2/$rigidSlope] $Vi3 [expr $Vi3/$rigidSlope]**
**[expr -$Vi1] [expr -$Vi1/$rigidSlope] [expr -$Vi2] [expr -$Vi2/$rigidSlope]\ [expr -$Vi3] [expr -$Vi3/$rigidSlope]**
**0.5 0.4 0.0 0.0 0.4 $shearCurveTag 2 0**

**Axial limit curve:**

AxialElasticSlope = 470080 Kips/Inch ($99.0E_cA/L$, Where $E_c$ is the modulus of elasticity of concrete, $A$ is gross cross-sectional area and $L$ is the height of column. ).

If the beam-column element includes the axial flexibility of the column, the pre-failure backbone for the axial spring should be defined by a steep straight line to ensure that the axial spring does not add axial flexibility to the model. If, on the other hand, the beam-column element is considered axially rigid, then the slope of the pre-failure backbone for the axial spring can be set equal to the initial axial stiffness of the column.

Axial loads used for setting the initial elastic slope are:
P1 = 65.0 Kips
P2 = 75.0 Kips
P3 = 85.0 Kips

To develop the axial spring response, the axial forces *Pi* (*i*=1 to 3) need to be assigned values in the increasing order as they all lay on the same line. It is also important that the last axial force, *Pi3*, is greater than the initial axial force, $P_{initial}$ . Even if the value of *Pi3* is significantly higher than $P_{initial}$, the results of the analysis will not change. In this example the value of $P3/P_{initial}$ is 85/70 = 1.21.

In this example:
**uniaxialMaterial LimitState $axialFailTag**
**$P1 [expr $P1/$axialElasticSlope] $P2 [expr $P2/$axialElasticSlope] $P3\\ [expr $P3/$axialElasticSlope] [expr -$P1] [expr -$P1/$axialElasticSlope] [expr -$P2]\\ [expr -$P2/$axialElasticSlope] [expr -$P3] [expr -$P3/$axialElasticSlope]**
**0.5 0.5 0.0 0.0 0.0 $axialCurveTag 1**